SPEED SOLUTION

Question 1

The bus has to cross not only a 60 foot gap but also another distance equal to the bus itself. The total horizontal distance for the jump is:

$$60 + 50 = 110ft$$

Now convert that into metres:

$$110\ ft\ =\ 110\ ft\ x\ 0.3048\ m\ /\ ft$$

$$110\ ft\ =\ 33.53\ m$$

We can convert the speed of the bus into km/hr and then m/s:

$$Bus\ speed\ =\ 70\ miles\ per\ hour\ (mph)$$

$$Bus\ speed\ =\ 70\ miles/hr\ \times\ 1.609\ km/m$$

$$Bus\ speed\ =\ 112.63\ km/hr$$

$$Bus\ speed\ =\ 112.63\ km/hr\ \times 1000\ m\ /\ km\ /\ (3600\ s\ /\ hr)$$

$$Bus\ speed\ =\ 31.29\ m/s$$ ***(1 mark)***

To find how far the bus will fall, we can use the following formula:

$$h\ =\ 0.5\ \times\ a\ \times\ t^2$$

$$h\ is\ the\ vertical\ fall\ distance, a\ is\ acceleration\ (in\ this\ case\ gravity, 9.81\ m/s/s)\ and\ t\ is\ the\ time\ duration\ of\ the\ fall$$

Next we can find the time duration of the fall using the formula:

$$jump\ time\ duration\ =\ distance\ /\ speed$$

$$jump\ time\ duration\ =\ 33.53\ /\ 31.29$$

$$jump\ time\ duration\ =\ 1.07\ s$$ ***(1 mark)***

Now we can substitute this jump duration time t into the fall formula:

$$h\ =\ 0.5\ \times\ a\ \times\ t^2$$

$$h\ =\ 0.5\ \times\ 9.81\ \times\ 1.07^2$$

$$h = 5.62\ m$$ ***(1 mark)***

We can conclude that the bus will have fallen a vertical distance of about 6 metres in crossing the gap.

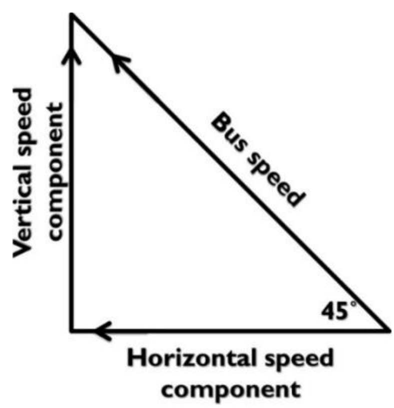
Question 2

From the first question we know that the front of the bus has to travel 110 ft, or 33.53 m.

Now, normally pitching upwards at 45 degrees would take a lot of speed off, but we'll assume for simplicity that the bus retains its full speed, but now moving upwards at 45 degrees.

We need to find the vertical component of that speed, in order to work out how long that bus will fly upwards, and then downwards before ending back at the same vertical level.

We can use the sine function:



$$sin(45) = opposite / hypotenuse$$

$$sin(45) = vertical\ speed\ component / bus\ speed$$

$$vertical\ speed\ component = bus\ speed\ x\ sin(45)$$

$$vertical\ speed\ component = 112.63\ km/hr\ x\ sin(45)$$

$$vertical\ speed\ component = 79.64\ km/hr$$ ***(1 mark)***

For the first half of the jump, the bus will be rising. Then it will pause, with zero vertical speed. For the second half, it will be falling.

Now we can calculate how long it will take for the bus's vertical speed component to drop from 79.64 km/hr to zero at the midpoint of the jump:

$$time\ to\ reach\ highest\ point\ of\ jump = initial\ vertical\ speed / gravity$$

$$time\ to\ reach\ highest\ point\ of\ jump = 79.64\ km/hr / 9.81\ m/s/s$$

$$time\ to\ reach\ highest\ point\ of\ jump = 79.64\ km/hr\ x\ 1000\ m/km / (3600\ s / hr) / 9.81\ m/s/s$$

$$time\ to\ reach\ highest\ point\ of\ jump = 2.26\ s$$

That makes up the first half of the jump, the rising part, so total jump duration will be:

$$Total\ jump\ duration\ =\ 2 \times 2.26s$$

$$Total\ jump\ duration\ =\ 4.52\ s$$ ***(1 mark)***

Now we can find how far the bus will travel horizontally during this time:

$$Horizontal\ travel\ distance\ =\ horizontal\ speed\ component \times jump\ duration$$

The triangle diagram shows that the horizontal speed component should be exactly the same as the vertical speed component:

$$Horizontal\ travel\ distance\ =\ horizontal\ speed\ component\ x\ jump\ duration$$

$$Horizontal\ travel\ distance\ =\ 22.12\ m/s \times 4.52\ s$$

$$Horizontal\ travel\ distance\ =\ 99.99\ m$$

So it would theoretically be possible for the bus to cross the gap and land on an equally high section of highway:

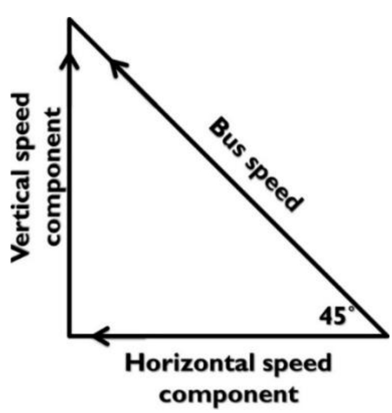
$$99.99m > 33.53m$$ ***(1 mark)***

Question 3

From the first question we know that the front of the bus has to travel the distance of 110 ft, or 33.53 m.

We again need to find the vertical component of that speed, in order to work out how long that bus will fly upwards, and then downwards before ending back at the same vertical level. In the first question we calculated that the speed of the bus is 112.63km/hr.

Now we use trigonometry, the sine function:



$$sin(30)\ =\ opposite\ /\ hypotenuse$$

$$sin(30)\ =\ vertical\ speed\ component\ /\ bus\ speed$$

$$vertical\ speed\ component\ =\ bus\ speed\ x\ \ sin(30)$$

$$vertical\ speed\ component\ =\ 112.63\ km/hr\ x\ \ sin(30)$$

$$vertical\ speed\ component\ =\ 56.32\ km/hr$$ ***(1 mark)***

You can calculate how long it will take for the bus's vertical speed component to drop from 56.32 km/hr to zero at the midpoint of the jump:

$$time\ to\ reach\ highest\ point\ of\ jump\ =\ initial\ vertical\ speed\ /\ gravity$$

$$time\ to\ reach\ highest\ point\ of\ jump\ =\ 56.32\ km/hr\ /\ 9.81\ m/s/s$$

$$time\ to\ reach\ highest\ point\ of\ jump\ =\ 56.32\ km/hr\ \times\ 1000\ m/km\ /\ (3600\ s\ /\ hr)\ /\ 9.81\ m/s/s$$

$$time\ to\ reach\ highest\ point\ of\ jump\ =\ 1.595\ s$$

That makes up the first half of the jump, the rising part, so the total jump duration will be:

$$Total\ jump\ duration\ =\ 2\ \times\ 1.595s$$

$$Total\ jump\ duration\ =\ 3.19\ s$$ ***(1 mark)***

Now find how far the bus will travel horizontally during this time:

$$Horizontal\ travel\ distance\ =\ horizontal\ speed\ component\ \times\ jump\ duration$$

From the triangle diagram we can find the horizontal speed component:

$$cos(30) = adjacent/hypotenuse$$

$$cos(30)\ = horizontal\ speed\ component\ /\ bus\ speed$$

$$horizontal\ speed\ component\ =\ bus\ speed\ x\ cos(30)$$

$$horizontal\ speed\ component\ =\ 112.63\ km/hr\ x\ \ cos\ (30)$$

$$horizontal\ speed\ component\ =\ 97.54\ km/hr$$ ***(1 mark)***

The horizontal speed component in units of m/s is:

$$horizontal\ speed\ component\ = 97.54\ km/hr\ x\ 1000\ m/km\ /\ (3600\ s\ /\ hr)$$

$$horizontal\ speed\ component = 27.09\ m/s$$

Now find the horizontal travel distance using the formula:

$$Horizontal\ travel\ distance\ =\ horizontal\ speed\ component \times jump\ duration$$

$$Horizontal\ travel\ distance\ =\ 27.09\ m/s \times 3.19\ s$$

$$Horizontal\ travel\ distance\ =\ 86.4\ m$$

$$86.4m > 33.53m$$

From this inequality, theoretically it is possible for the bus to cross the gap and land on an equally high section of highway. **(1 mark)**