CAR CRASH INVESTIGATION WORKED SOLUTION

Question 1

The car speed can be calculated by dividing the distance the car travels between the two camera frames by the time interval between the two camera frames:

$$car\ speed = \frac{distance}{time}$$

$$car\ speed = \frac{25}{1}$$

$$\boldsymbol{car\ speed = 25\ m/s}$$ *(1 mark)*

To get the speed into familiar units of *km/hr*, we can multiply by 3600 seconds (in an hour) and divide by 1000 (1000 metres in a km):

$$car\ speed = \frac{25 \times 3600}{1000}$$

$$\boldsymbol{car\ speed = 90\frac{km}{hr}}$$ *(1 mark)*

Question 2

The gradient of a road is the distance the road *rises* (positive gradient) or falls (negative gradient) divided by the horizontal distance the road travels (the '*run*').

In the provided diagram, the road rises 10 metres over a horizontal distance of 150 metres. We can work out the gradient:

$$gradient = \frac{rise}{run}$$

$$gradient = \frac{10}{150}$$

$$\boldsymbol{gradient = 0.0667}$$ ***(1 mark)***

Question 3

The equation has 5 parameters or variables in it and we know all 5. So we can simply plug in the numbers and calculate the final car velocity (making sure we use consistent units):

$$v_{end} = \sqrt{v_{initial}^2 - 2 \times g \times d_{braking} \times (f \pm G)}$$
$$v_{end} = \sqrt{25^2 - 2 \times 9.81 \times 30 \times (0.7 + 0.0667)}$$
$$v_{end} = 13.18 m/s$$
$$v_{end} = 47.44 km/hr$$

(1 mark)

Question 4

The question asks us to calculate the car speed that results in the same stopping distance on a downhill road as a car travelling at 80 km/hr on a flat road.

We immediately know that the braking distance $d_{braking}$ needs to the same for the flat road case and the downhill case. We can calculate the actual braking distance for the 80 km/hr case:

$$0 = \sqrt{v_{initial}^2 - 2 \times g \times d_{braking} \times (f \pm G)}$$
$$0 = \sqrt{22.22^2 - 2 \times 9.81 \times d_{braking} \times (0.7 + 0.0)}$$
$$2 \times 9.81 \times d_{braking} \times (0.7 + 0) = 22.22^2$$
$$d_{braking} = 35.95 m$$

(1 mark)

We also know that the end velocity v_{end} needs to be zero in both cases, because this represents the car slowing to a complete stop. We can write two versions of the equation, one for each scenario, and *equate* them:

$$v_{\text{end flat}} = v_{\text{end downhill}} = 0$$
$$\sqrt{v_{\text{initial flat}}^2 - 2 \times g \times d_{braking} \times (f + 0)} = \sqrt{v_{\text{initial downhill}}^2 - 2 \times g \times d_{braking} \times (f - 0.1)}$$

(1 mark)

We can get rid of the square root signs by squaring both sides:

$$\sqrt{v_{\text{initial flat}}^2 - 2 \times g \times d_{braking} \times (f + 0)} = \sqrt{v_{\text{initial downhill}}^2 - 2 \times g \times d_{braking} \times (f - 0.1)}$$
$$v_{\text{initial flat}}^2 - 2 \times g \times d_{braking} \times (f + 0) = v_{\text{initial downhill}}^2 - 2 \times g \times d_{braking} \times (f - 0.1)$$

Then we can re-arrange the equations to get the downhill initial velocity variable on one side and everything else on the other side.

$$v_{\text{initial flat}}^2 - 2 \times g \times d_{braking} \times (f + 0) = v_{\text{initial downhill}}^2 - 2 \times g \times d_{braking} \times (f - 0.1)$$
$$v_{\text{initial flat}}^2 - 2 \times g \times d_{braking} \times (f + 0) + 2 \times g \times d_{braking} \times (f - 0.1) = v_{\text{initial downhill}}^2$$
$$v_{\text{initial downhill}} = \pm\sqrt{v_{\text{initial flat}}^2 - 2 \times g \times d_{braking} \times (f + 0) + 2 \times g \times d_{braking} \times (f - 0.1)}$$

(1 mark)

Now all we need to do is substitute in the actual values (using 80 km/hr converted into m/s to be consistent with the gravity value being given in m/s/s) and the $d_{braking}$ value we calculated earlier:

$$v_{\text{initial downhill}} = \pm\sqrt{v_{\text{initial flat}}^2 - 2\times g\times d_{braking}\times(f+0) + 2\times g\times d_{braking}\times(f-0.1)}$$
$$v_{\text{initial downhill}} = \pm\sqrt{22.22^2 + 2\times g\times d_{braking}[f-0.1-f]}$$
$$v_{\text{initial downhill}} = \pm\sqrt{22.22^2 + 2\times g\times 35.95\times -0.1}$$
$$v_{\text{initial downhill}} = \pm 20.57 m/s$$
$$v_{\text{initial downhill}} = 74.06 km/hr$$

(1 mark)

Although both answers make mathematical sense, I've picked the positive answer given the context of the problem. So travelling at 74 km/hr on a moderate downhill slope requires the same stopping distance as travelling at 80 km/hr on a flat road, all other things being equal.